# EVALUATION OF SOLAR CELL J(V)-MEASUREMENTS WITH A DISTRIBUTED SERIES RESISTANCE MODEL


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ABSTRACT: Least square fits of *J(V)*-curves to the double diode model often result in different parameters for the illuminated and dark characteristics which are not compatible with the $J_{SC}$-$V_{OC}$ curve, especially for non optimum solar cells in the development stage. When using a one-dimensional distributed series resistance model the illuminated, dark *J(V)* and $J_{SC}$-$V_{OC}$ characteristics of many of our solar cells can be well described with a consistent set of parameters, i.e. no light dependent parameters for series resistances and diodes need to be assumed. Rather then fitting all parameters to a single curve, we extract the parameters $R_{SH}$, $J_{01}$, $J_{02}$ and the n-factors from the dark *J(V)*-curve and the $J_{SC}$-$V_{OC}$ curve, respectively. The illuminated curve then provides sufficient information to separate two contributions to the series resistance. In addition to the series resistance $R_S$, a parameter $R_{CC}$ (in Ωcm², *CC* for current crowding) is used to describe the distributed character of a *J(V)* curve independently from the cell geometry. The interpretation of the fitted resistance values are discussed as well as the tendency towards wrong results when distributed cell characteristics are fitted to the ordinary double diode model.



Keywords: Characterisation - 1: Evaluation - 2: Modelling - 3


## INTRODUCTION

The measurement of the current-voltage behaviour is the most obvious characterisation technique for solar cells. Measured *J(V)*-curves are mostly evaluated by a least square fit to the common equivalent circuit model consisting of a current source, one or two diodes, a shunt resistance and a series resistance. This double diode model assumes, that the voltage is homogeneous over the solar cell but reduced by the a voltage drop at the series resistance which is proportional to the device current. Most of our solar cell *J(V)*-curves can be described well with this model, but the resulting parameters are often very different for illuminated and dark characteristics. When comparing the dark characteristics with a calculated curve using the parameters found from the illuminated curve and vice versa the discrepancy becomes obvious as shown in Fig. 1. Here a cell with serious fill factor reduction was chosen to make the problem more obvious. A third set of diode parameters is obtained from a $J_{SC}$-$V_{OC}$-curve [1] which frequently appears incapable to reproduce dark and illuminated IV-curves with the double diode model as demonstrated in Fig.1 (bottom), where $R_S$ was chosen to match the maximum power point.

Possible reasons for these differences are the invalidity of the superposition principle due to injection dependent recombination parameters [2], spatial inhomogenities [3], enhanced recombination beneath the metal contacts in the dark case [4] and the failure of the double diode model because of the distributed nature of the series resistance. The latter is the dominant factor for most of our solar cells.

Many authors have already investigated this topic by either computer simulation studies about the behaviour of an distributed model [5-10]which is therefore well understood, or the interpretation of *J(V)*-curves with help of lumped series resistances [11]. Bell [12] had already found it necessary to fit to a distributed model when solar cells become large in area. In contrast to his method, where six parameters are fitted using all points of an illuminated curve, we find it necessary to use a dark, an illuminated and a $J_{SC}$-$V_{OC}$ curve to extract a consistent set of parameters. Furthermore, he assumed the series resistance to be completely distributed, which is insufficient to describe our cells (mostly large area screen printed Si solar cells).

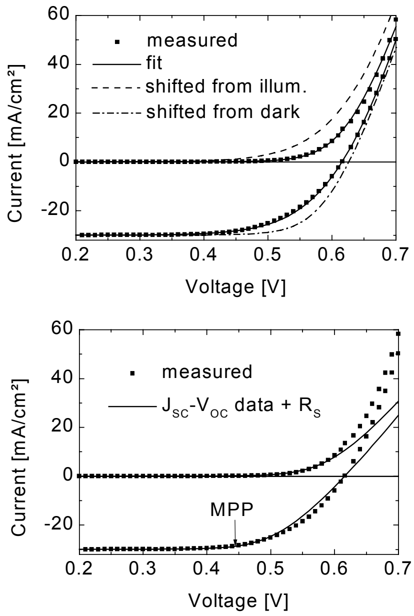
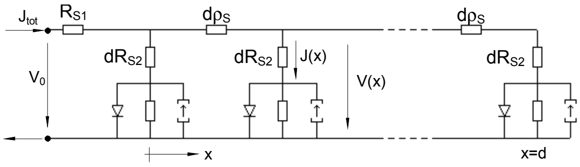



**Figure 1:** *Measured J-V-characteristics and least square fits of a 100cm² screen-printed Si solar cell. Top: Discrepancy of fit results when double diode model is inappropriate. Bottom: Discrepancy when using the results of $J_{SC}$-$V_{OC}$ data and a single series resistance.*

## DISTRIBUTED SERIES RESISTANCE MODEL




The equation describing the voltage distribution in a layer with sheet resistance $\rho_S$ is

$$V''(x) = J(V)\rho_S \qquad (1)$$

with the boundary conditions, that the voltage at $x = 0$ is $V_0$ and the current in the sheet must be zero at $x = d$. Using the normalised length $s = x/d$ eqn. 1 becomes

$$V''(s) = J(V)\rho_S d^2 \,. \qquad (2)$$

and the total current density is

$$J_{tot} = \int_0^1 J(s)ds \qquad (3)$$

*J(V)* is the local current density corresponding to the two-diode model either with (model B) [5] or without series resistance $R_{S2}$ (model A). In model A $R_{S1}$ is in series with the whole sheet causing a voltage shift by $J_{tot}\, R_{S1}$. Interestingly $\rho_S\, d^2$ can be regarded as a single parameter which we call here $R_{CC}$. It means, for instance, that a sheet with 100 Ω/sq and a length of 1 mm has the same *J(V)*-characteristics like a sheet with 1Ω/sq and 1cm length since both are giving an $R_{CC} = \rho_S d^2 = 1\Omega\text{cm}^2$.

Using one single series resistance proved to be insufficient to describe our *J(V)*-curves, but on the other hand there is not enough information to distinguish three contributions as would follow when combining model A and B. We solve the equations with a fourth order Runge-Kutta algorithm with step width adaption [13]. Although model B may in some cases be more appropriate to the actual solar cell, fitting to model A is much faster since the voltage shift $J\, R_{S1}$ can be subtracted from the experimental *J-V*-data instead of adding it to the calculated data*. The local *J(V)* for the two diode model without series resistance $R_{S2}$ can then be calculated explicitly which further simplifies the calculation for model A**. The two models can be used equally to describe the *J(V)*-curves unless the current densities are too high, where the distributed model is also questionable for other reasons [4].

At $V=V_{OC}$ the equations can be linearised providing an exact expression for $R_{OC}$, the inverse slope of the *J(V)*-curve at $V_{OC}$:

$$R_{OC} = R_{S1} + R_{CC}\sqrt{\frac{R_D + R_{S2}}{R_{CC}}}\coth\left(\sqrt{\frac{R_{CC}}{R_D + R_{S2}}}\right) \qquad (4)$$

$R_D$ is the inverse slope at $V_{OC}$ for the double diode model without any series resistance. Series expansion of eqn. 4 for $R_{CC} < R_D + R_{S2}$ suggests, that if both models shall give the same $R_{OC}$ then $R_{Si}+R_{CC}/3$ must have the same value*. A simple lumped series resistance which will match the maximum power point is also of the magnitude $R_{Si}$ + $R_{CC}/3$ within some error as will be discussed later.

## ERRORS IN PARAMETERS FITTED WITH THE DOUBLE DIODE MODEL

When fitting illuminated J-V-characteristics to a double-diode model the tendency towards wrong results is caused by the fact, that the distributed nature of the series resistance produces a rather round curve around the maximum power point. When fitting to the simple model the value for the second diode may be increased to describe this rounding as long as it is compatible with the actual value of $V_{OC}$. As a consequence the first diode usually dominating $V_{OC}$ is reduced to a smaller value to fit the actual value of $V_{OC}$. The range above $V_{OC}$ is therefore described by the less steep *J(V)*-behaviour of the second diode so that the series resistance needs to be reduced to adjust to the steeper slope of the measured curve. This may lead to doubtful values for all parameters.

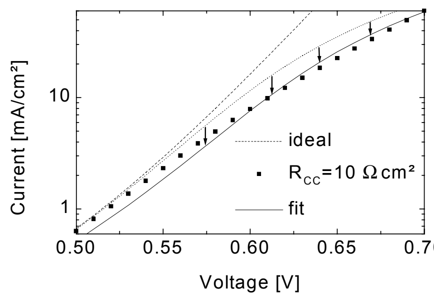



**Figure 2:** *Exaggerated example for the tendency towards underestimation in $J_{01}$ from fitting to inappropriate model.*

How errors in the results arise when fitting the dark-characteristics of a cell with distributed resistance to the double diode model is illustrated in Fig 2. Dark *J-V* curves of cells with purely distributed series resistance ($R_S$=0) show an asymptotical *J(V)*-behaviour towards $J \propto \exp(qV/2nkT)$ for high currents [9] whereas the double diode model ($R_{CC} = 0$) predicts linear behaviour. In order to better describe the high current range with a single series resistance the first diode saturation current will be reduced. In the lower current range this will be partly compensated by an increase in the second diode. The fitted series resistance will generally be lower than the meaningful $R_{S,i}$ + $R_{CC}/3$ which would be fitted only in the very low current range [5].

* The resulting voltage after adding $J\, R_{S1}$ cannot be predicted so that several iterations would be necessary to find the current density to a given voltage.

** The implicitly given local *J(V)* for $R_{S2}$ <>0 has to be found iteratively.

* This corresponds to a uniform current distribution, where the series resistance can also be calculated via the Joule heating losses in the resistive components [7].

## FITTING PROCEDURE

Since fitting with the distributed model is rather tedious it should be avoided as far as possible. Therefore all parameters other than the series resistances should be determined using the simple model. However, the conclusion of the previous section is, that fitting to an inappropriate model requires care and leads us to the following procedure to minimise errors:

1. Fitting the shunt resistance and second diode parameters from the dark curve where an estimated first diode saturation current is held fixed and all data points affected by an estimated series resistance are disregarded.
2. Extracting the saturation current of the first diode from the $J_{SC}$-$V_{OC}$ curve using shunt and second diode found in the dark curve. (If non is available then $J_{01}$ is chosen to match $V_{OC}$.)
3. Fitting the illuminated characteristics to the distributed model to obtain series resistances $R_S$ and $R_{CC}$ and compare if the results also describe the dark curve. Optionally illuminated and dark curves are fitted simultaneously.
4. Checking if the $J_{SC}$-$V_{OC}$-curve is valid i.e. if series resistance does affect $J_{SC}$.

If the normal double diode model is used in 3) then the series resistance is chosen to match the maximum power point, ignoring that this might only poorly describe the dark and illuminated curves.

## EXPERIMENTAL

To measure the $J_{SC}$-$V_{OC}$ curve we use the decay of a 500W halogen lamp to sweep through the illumination intensity. The decay time constant is about 100ms so that the cell heating can be kept small and the measurement is done within a few seconds. The intensity ($I_L$) is measured by a reference photo diode. We measure $J_{SC}(I_L)$ and subsequently $V_{OC}(I_L)$ and then calculate $J_{SC}$ vs. $V_{OC}$. It is important to take the dark, illuminated and $J_{SC}$-$V_{OC}$-curves at the same temperature, which we measure directly on the rear of the cell. It is furthermore best to measure all three curves with the same contact setting to avoid altering the resistances. To make evaluation easier the contacts should be placed either in the middle or at the end of the busbar to have the unit cell defined.

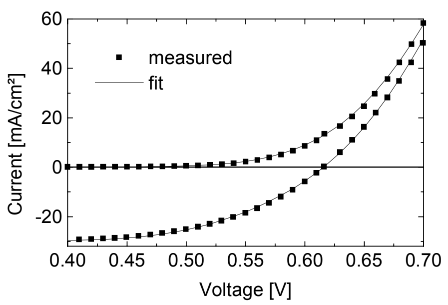



**Figure 3:** *Measured dark and illuminated IV-characteristics and least square fits to the distributed model (A)*

The distributed model was implemented in a double diode least square fit-program that runs on a PC. The time required for the evaluation is acceptable as all parameters but the series resistances are fitted to the double diode model. Fitting the illuminated curve to the distributed model is typically done in less than one minute since there are only two free parameters and since good starting values can be found using $R_{OC}$ from eqn. 4 and the maximum power point in a pre-fit.

Fig. 3 shows the same cell as in Fig.1. but evaluated by model A. All three curves can be described with a consistent set of parameters. The results are listed in Table 1 together with the inconsistent results from the normal fits.

## INTERPRETATION

Interpretation of the fitted series resistance $R_{CC}$ is simple if one of its sources dominates. If the source is the resistance in the emitter or a TCO-coating then $R_{CC}$ is just the sheet resistance $\sigma_S$ times its squared length $d$ of a unit cell. A definition for a unit cell can be found e.g. in [9].

$$R_{CC} = \rho_S d^2 \quad (5)$$

If the source is the resistance within the metal grid the equivalent sheet resistance has to be calculated. If the finger spacing is $W$ as in Fig. 4. and the finger resistance $R_F$ (in Ω/cm) then $\rho_S = R_F W$.

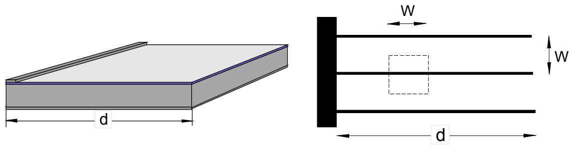



**Figure 4:** *Geometry for calculating $R_{CC}$: Emitter/TCO (left), metal contact grid (right).*

## SIMULATION

In order to find out how to interpret the results if contributions from emitter, finger and busbar are all important we performed a "quasi multi-dimensional" simulation by first calculating the $J$-$V$-behaviour of an emitter element, using the result as local relation for $J(V)$ along the finger and this result in turn along the busbar. The final $J(V)$ curve was then fitted to model A and B, respectively. This is of course not exact as the current collected by the emitter close to the busbar will flow directly to the busbar rather than into the finger. For a finger spacing much smaller than the finger length, however, this effect should be small.

The parameters were chosen as $J_{SC}$ = 30 mA/cm², $J_{01}$ = 1pA/cm² giving a $V_{OC}$ of 620mV. $R_S$ was set to zero here. We considered two cases: a) the same $R_{CC}$ used for each iteration ($R_{CC}$ = 0.3, 1 and 3 Ωcm²) and b) the set ($R_{CC}$ = 0.3, 1, 3 Ωcm²) applied in different orders. The following conclusions could be drawn from this empirical study:

1) $R_{CC} + 3\ R_S$ for both models equals the sum of $R_{CC}$-values put into the calculation.
2) Any order of the combination ($R_{CC}$ = 0.3, 1, 3 Ωcm²) results in the same fill factor (76.5 %).
3) When the same value $R_{CC}$ is used again and again the distributed character decreases after each iteration. We quantify this distributed character by $X = R_{CC} / (R_{CC} + 3\ R_S)$. $X$ is found empirically to be $1/(n/2+1)$ after $n$ iterations with model A and $1/(n/6+1)$ for model B.

**Table 1:** *Results of least square fits of the J(V)-curves of a 100cm² screen-printed Si solar cell.*

| | Dark: Normal fit | illum.: normal fit | illum.: $J_{SC}$-$V_{OC}$ *and* $R_{S,lumped}$ | illum. + dark (Model B) | illum. + dark (Model A) |
|---|---|---|---|---|---|
| $J_{01}$ [pA/cm²] | 0.69 | - | 1.09 | 1.09 | 1.09 |
| $J_{02}$ [nA/cm²] | 17.4 | 188 | 14.5 | 14.5 | 14.5 |
| $R_S$ [Ωcm²] | 1.03 | 0.68 | 2.87 | 0.1 | 0.31 |
| $R_{CC}$ [Ωcm²] | - | - | - | 6.8 | 5.85 |

4) The distributed character is strongest if the dominating $R_{CC}$ is caused by the emitter and weakest if caused by the busbar. (See Table 2)

**Table 2:** *Distributed character for different orders of* $R_{CC}$ *in emitter, finger and busbar.*

| order | *X* (model A) |
|---|---|
| 3 / 0.3 / 1 | 74 % |
| 3 / 1 / 0.3 | 75 % |
| 1 / 3 / 0.3 | 64 % |
| 0.3 / 3 / 1 | 59 % |
| 1 / 0.3 / 3 | 59 % |
| 0.3 / 1 / 3 | 55 % |

From 1) , 2) and 3) we conclude that it is useful to replace the two series resistance parameters by the already familiar lumped series resistance $R_{S,lumped}$ and the distributed character $X_i$ (index *i* for the used model):

$$\begin{matrix} R_{S,i} & | \\ R_{CC,i} & | \end{matrix} \Leftrightarrow \begin{cases} R_{S,lumped} = R_{S,i} + R_{CC,i} / 3 \\ X_i = (R_{CC,i} / 3) / R_{S,lumped} \end{cases}$$

This lumped series resistance put into the common double diode model will also describe the maximum power point with minor error if $R_{S,lumped}$ it is not too high, as is illustrated in Fig. 5.

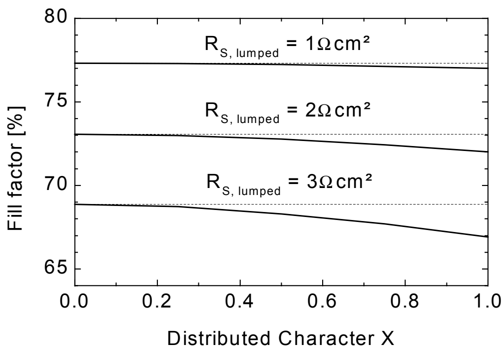



**Figure 5:** *Validity of the lumped series resistance concept to describe the fill factor (model B used)*

From 2) a rule for the relation for the distributed character is provided between the models A and B: $X_A=X_B/(3-2 X_B)$ This relation depends a little on the voltage range (for the simulations we used 0 to 700mV).

Introducing $R_{CC}$ offers a degree of freedom capable to describe dark and illuminated curves with the diode parameters found from the $J_{SC}$-$V_{OC}$ curves, but this might also be accomplished by other models. As the models used in this work basically consider a voltage distribution that is caused by a sheet resistance, a spatial distribution of contact resistance, for instance, can lead to a similar voltage distribution over the pn-junction and would therefore give similar results. In addition the used distributed models will become inappropriate for very strong current crowding, e.g. at high voltages the current will mainly flow directly from the metal finger into the base rather than being crowded over a small length in the emitter.

## CONCLUSIONS

Using the distributed model allows us to actually reduce the number of parameters needed to reproduce the dark and illuminated *J(V)* and $J_{SC}$-$V_{OC}$ curve by introducing the additional parameter $R_{CC}$. No artificial light dependent parameters need to be assumed. $R_{CC}$ has a direct physical meaning when a single source e.g. the busbar-resistance dominates. The experience with the behaviour of the distributed model led us to a fitting procedure which is more time consuming but gives consistent results.

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